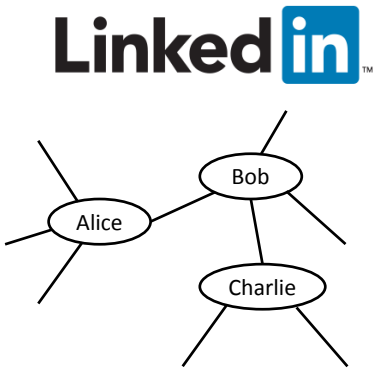
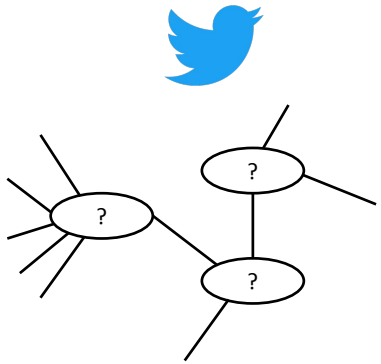
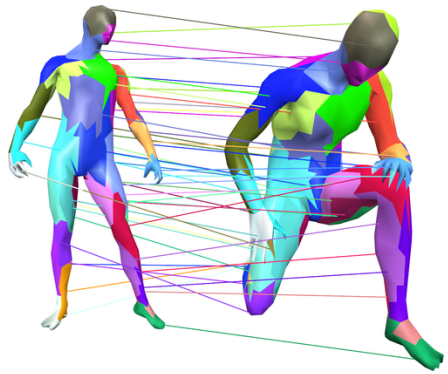
Graph Matching for Stochastic Block Models

Jian Ding, Peking University

with Guanyi Chen, Shuyang Gong, Zhangsong Li







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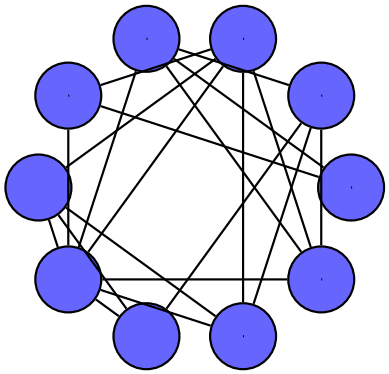

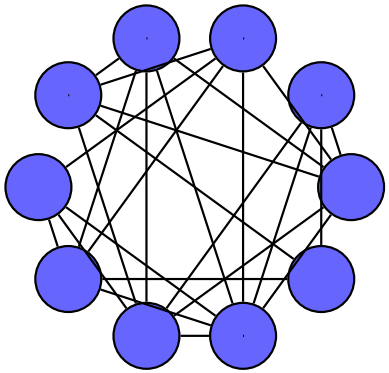

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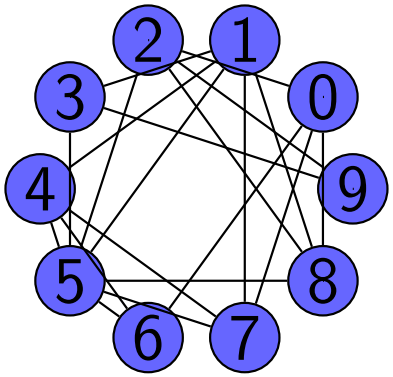

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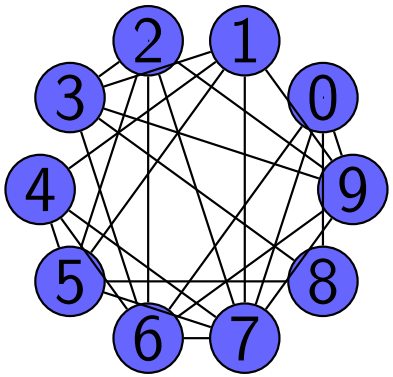

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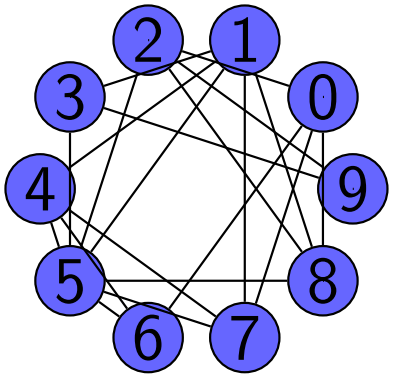

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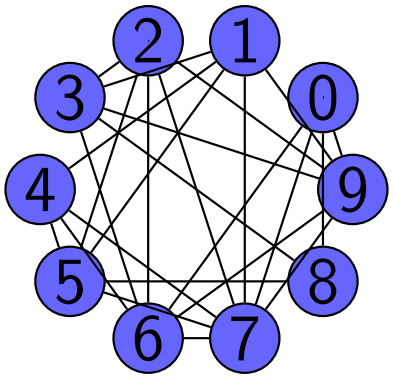

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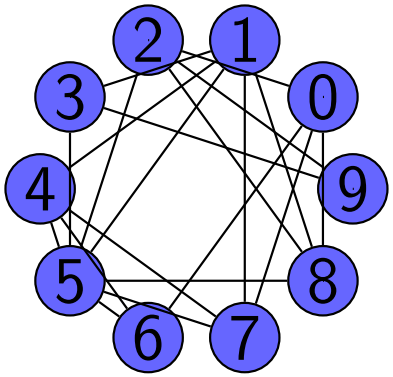

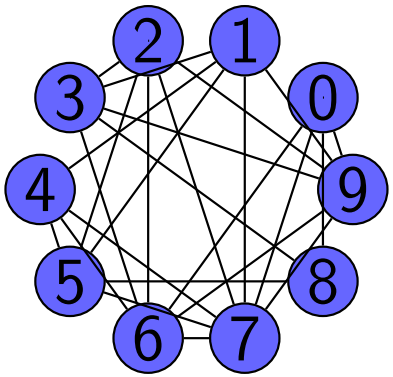

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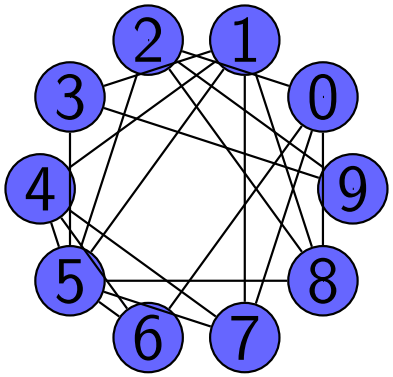

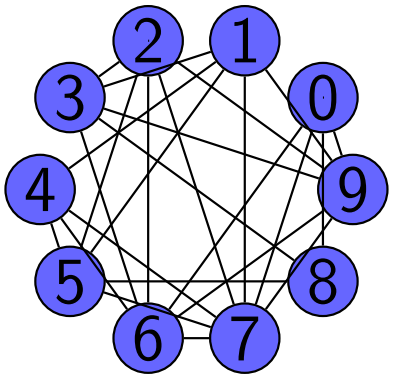

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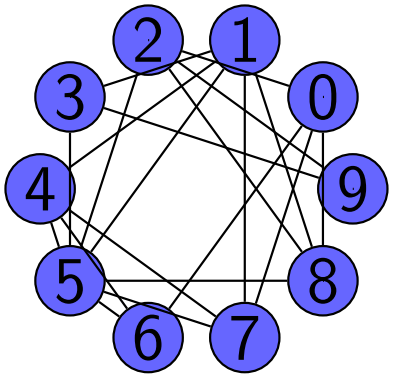

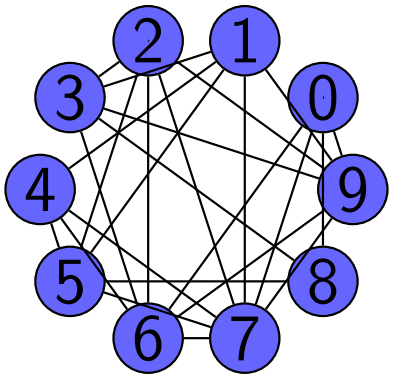

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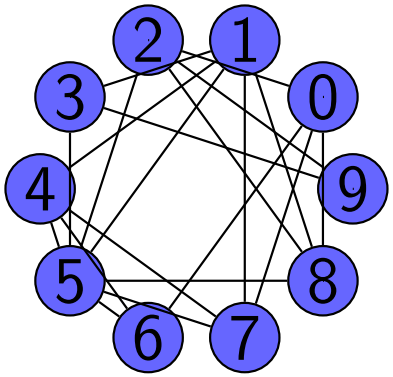

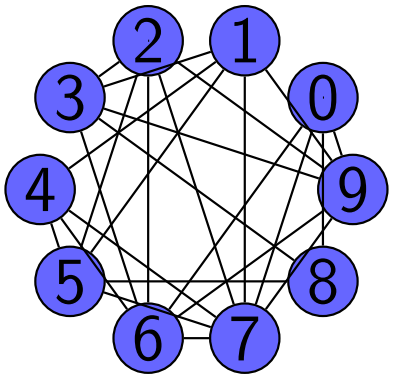

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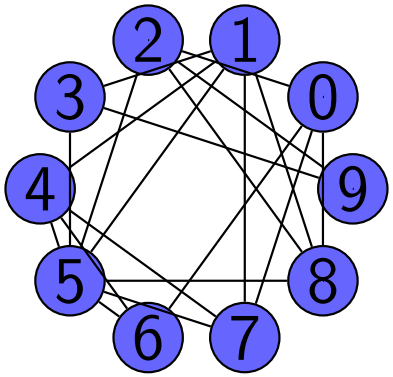

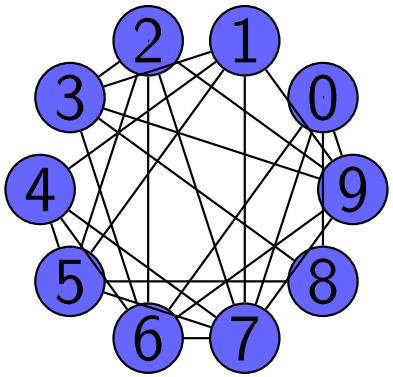

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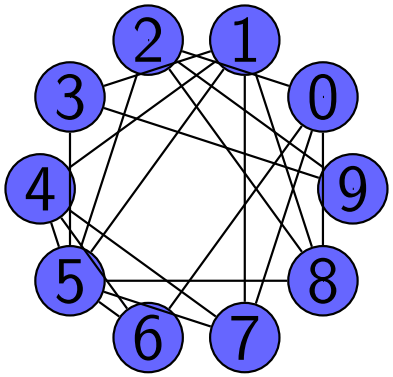

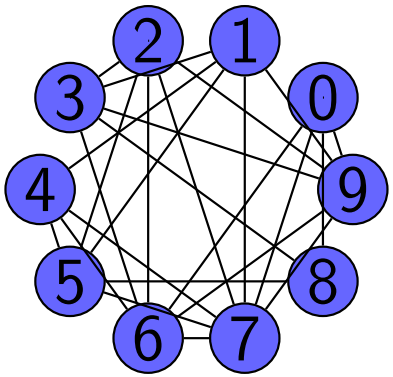

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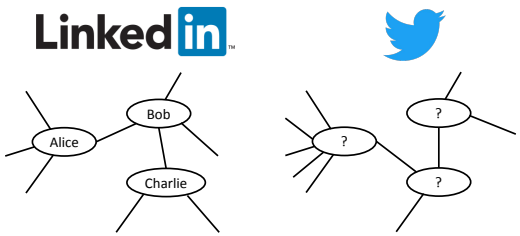

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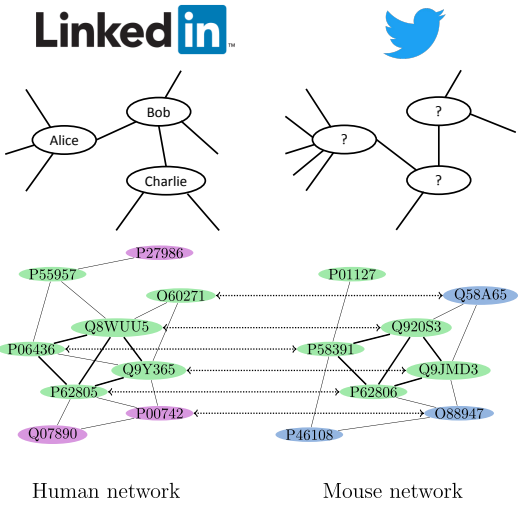

Human network Mouse network

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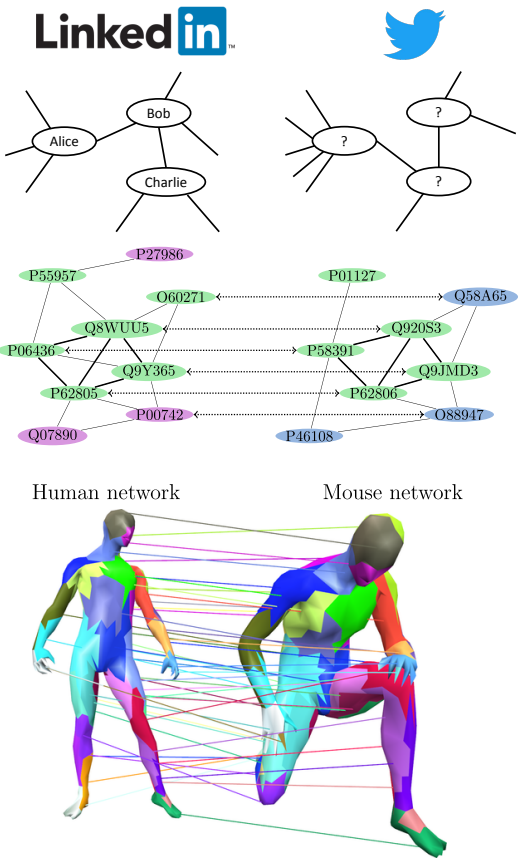

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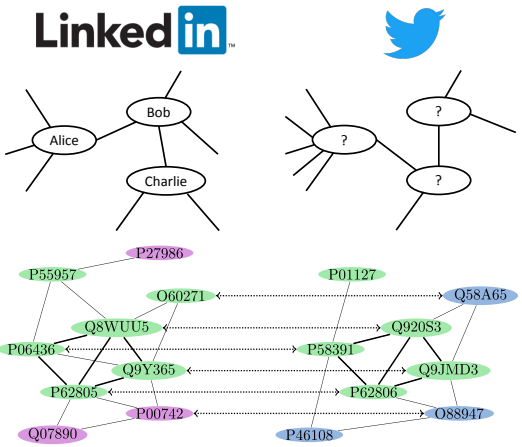

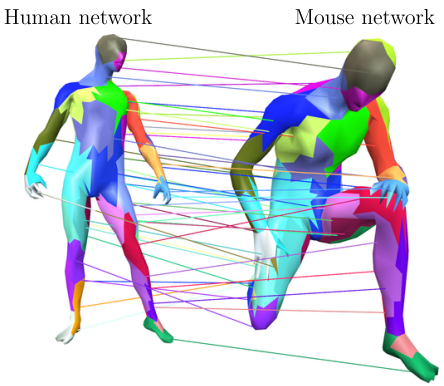

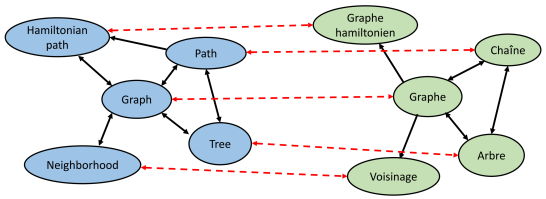

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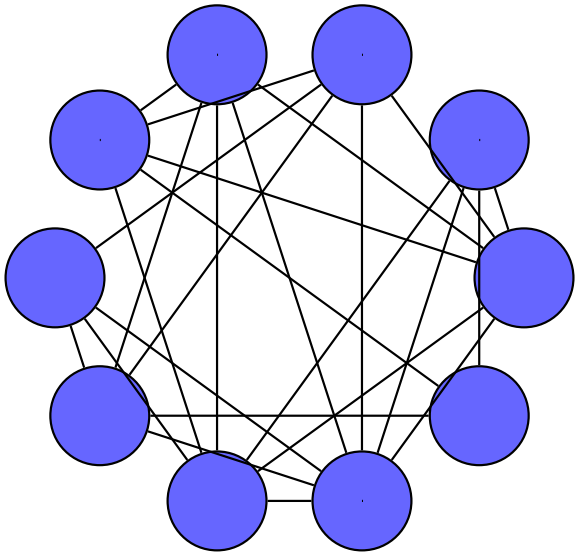
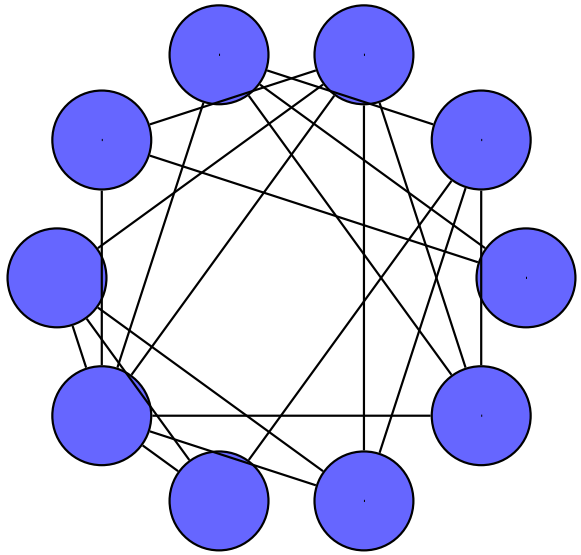
- Machine Translation: automatically find correct corresponding wiki articles in different languages in Fishkind-Adali-Patsolic-Meng-Lyzinski-Priebe '12.

Mathematical formulation of graph matching





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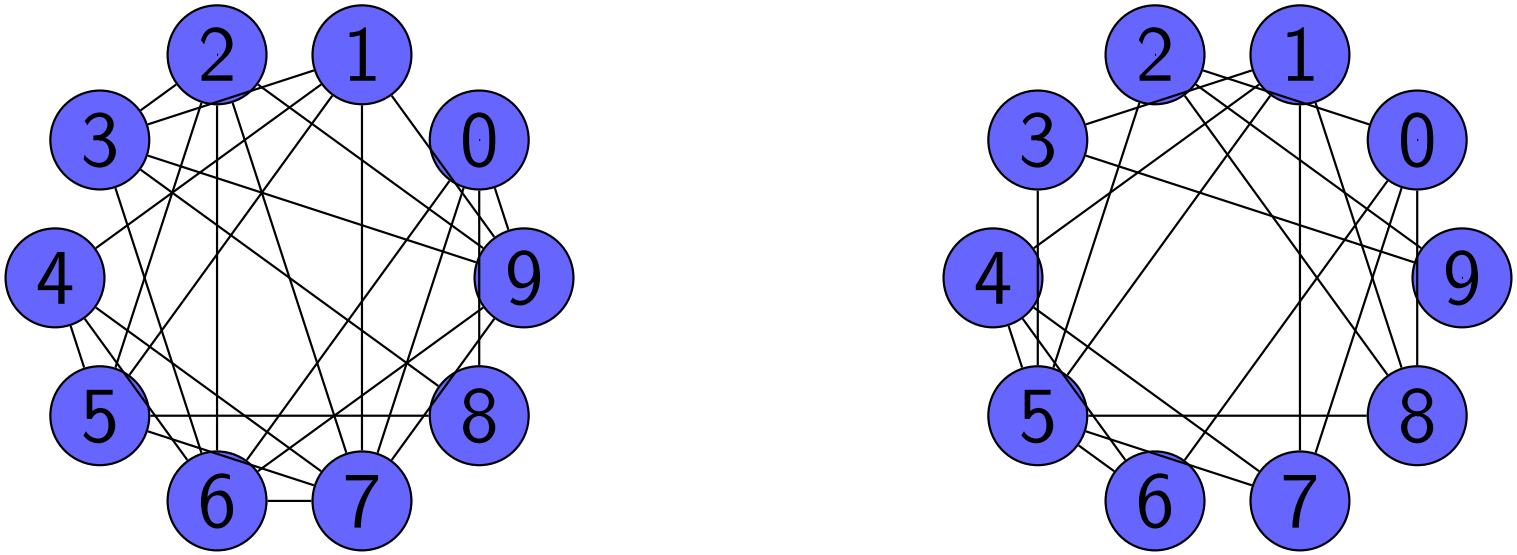

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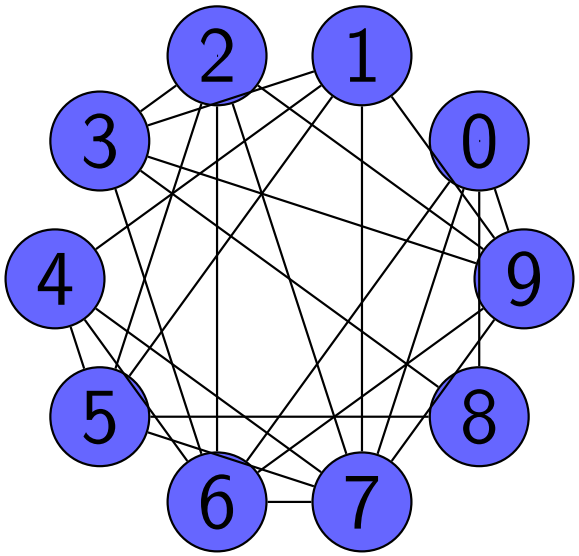

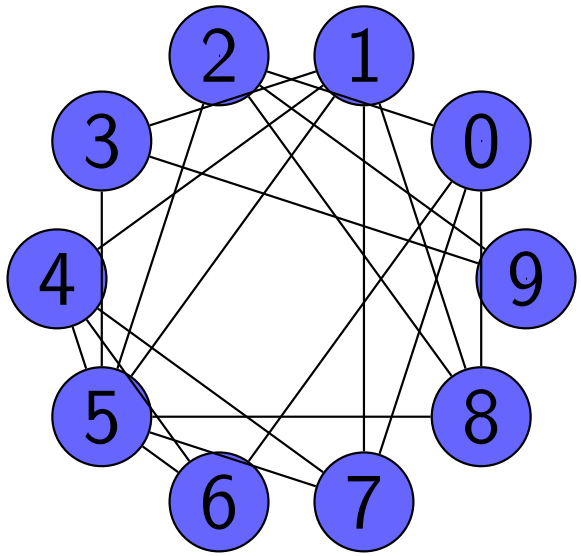

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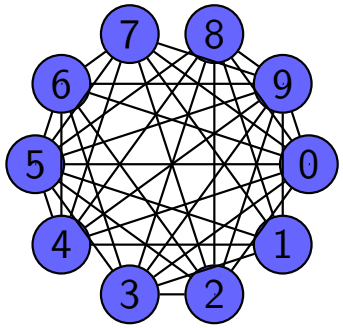
• Since graph matching is a hard optimization problem (NP-hard), we seek help from randomness.

An idealized model: Correlated Erdős-Rényi graphs model



$G_0 \sim \mathcal{G}(n, p)$

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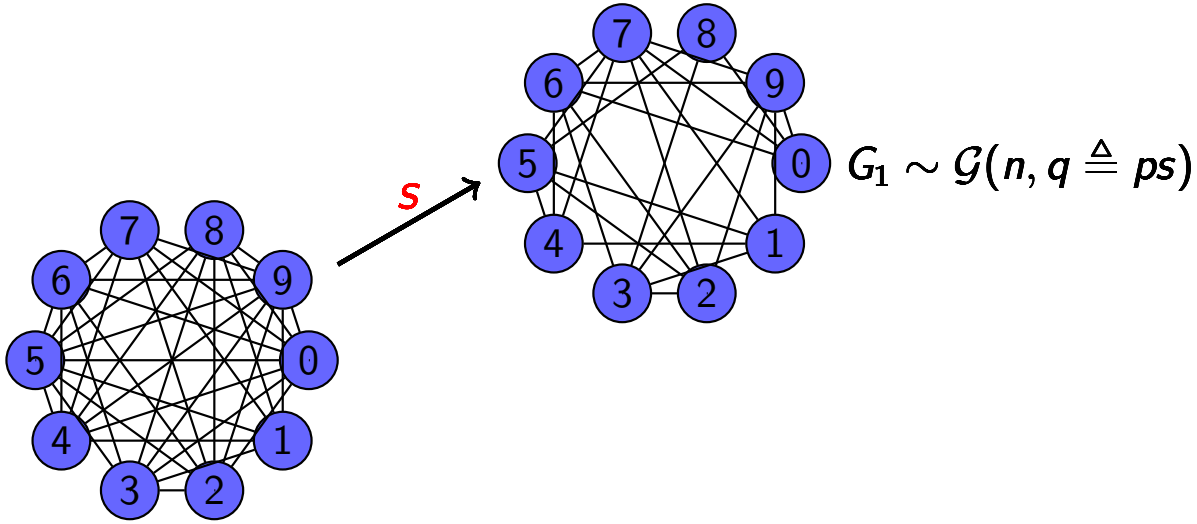

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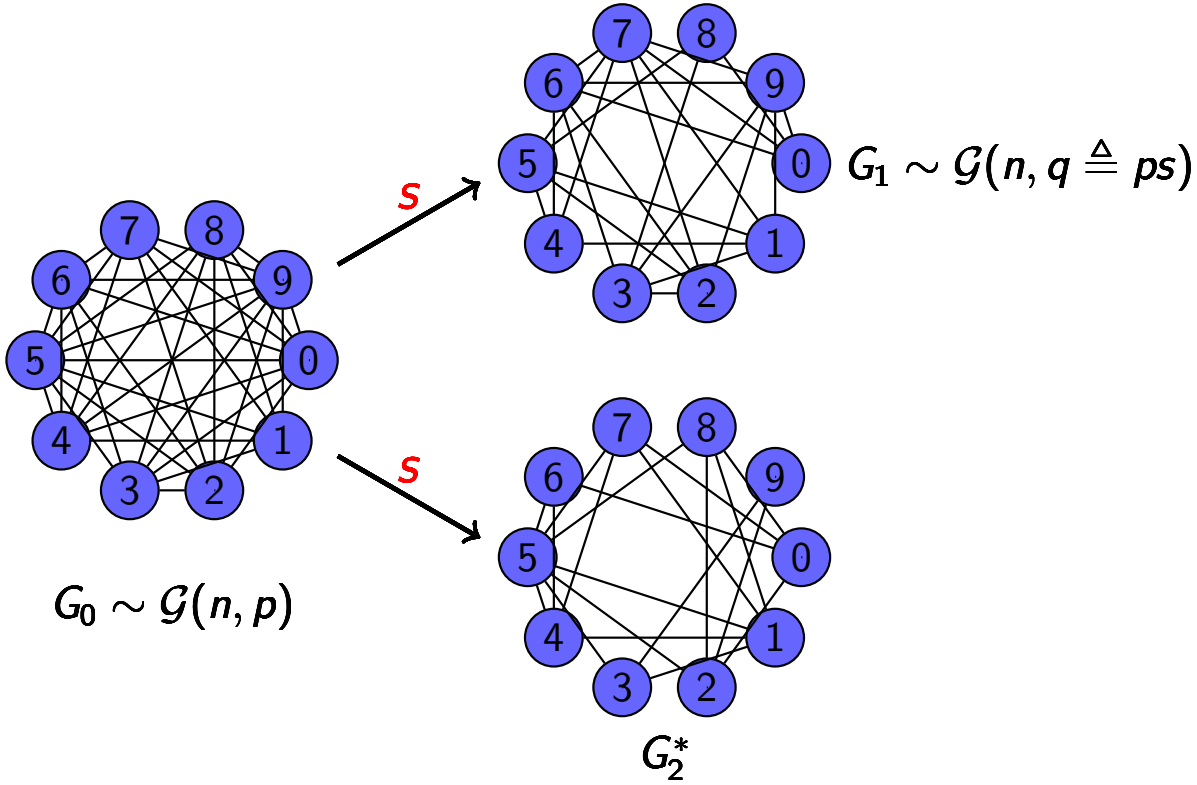

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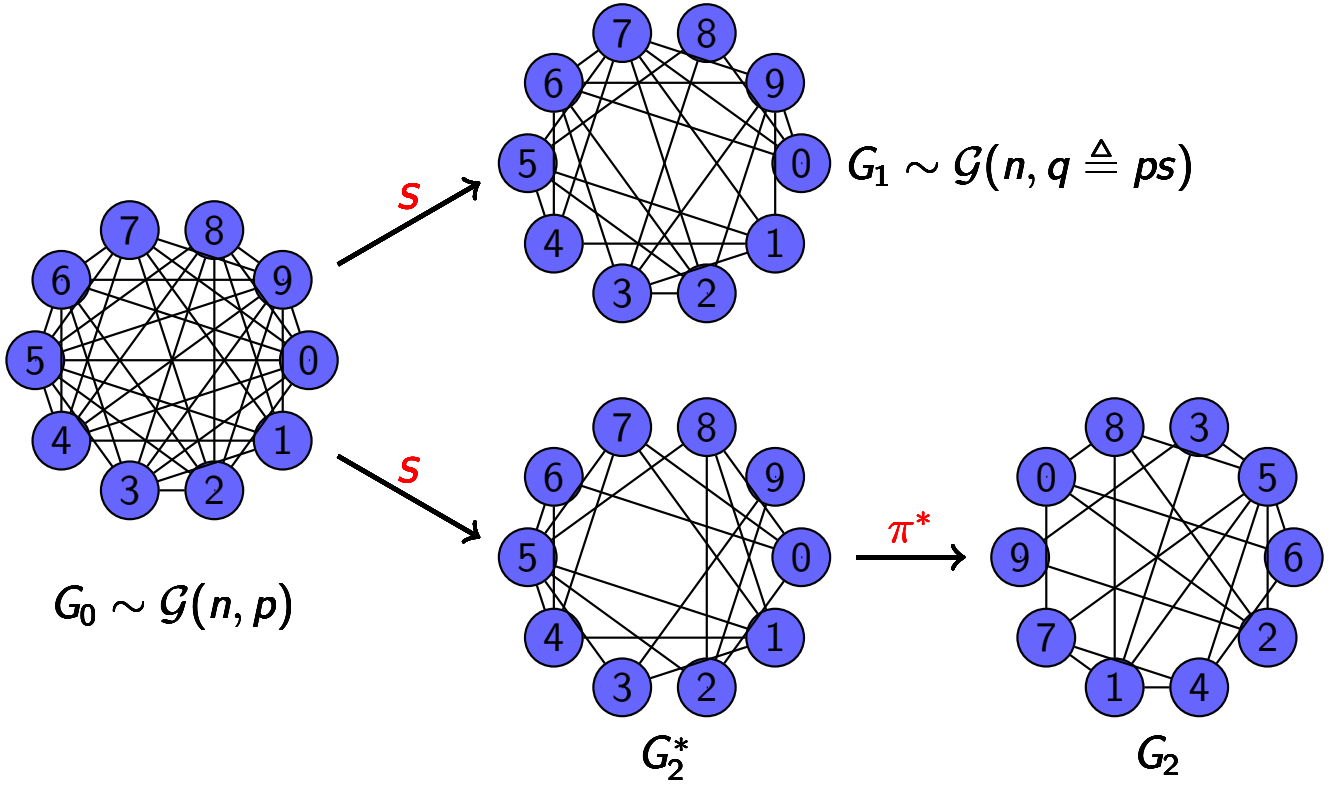

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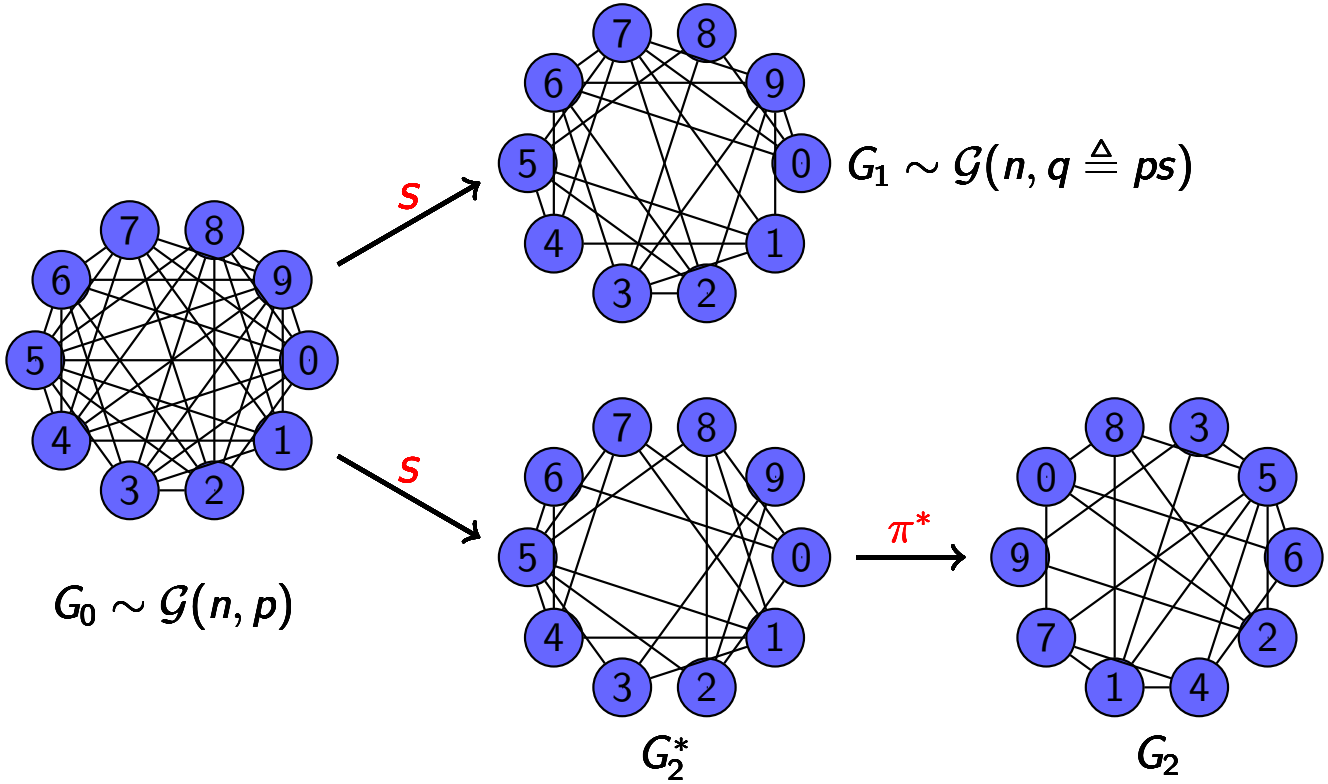

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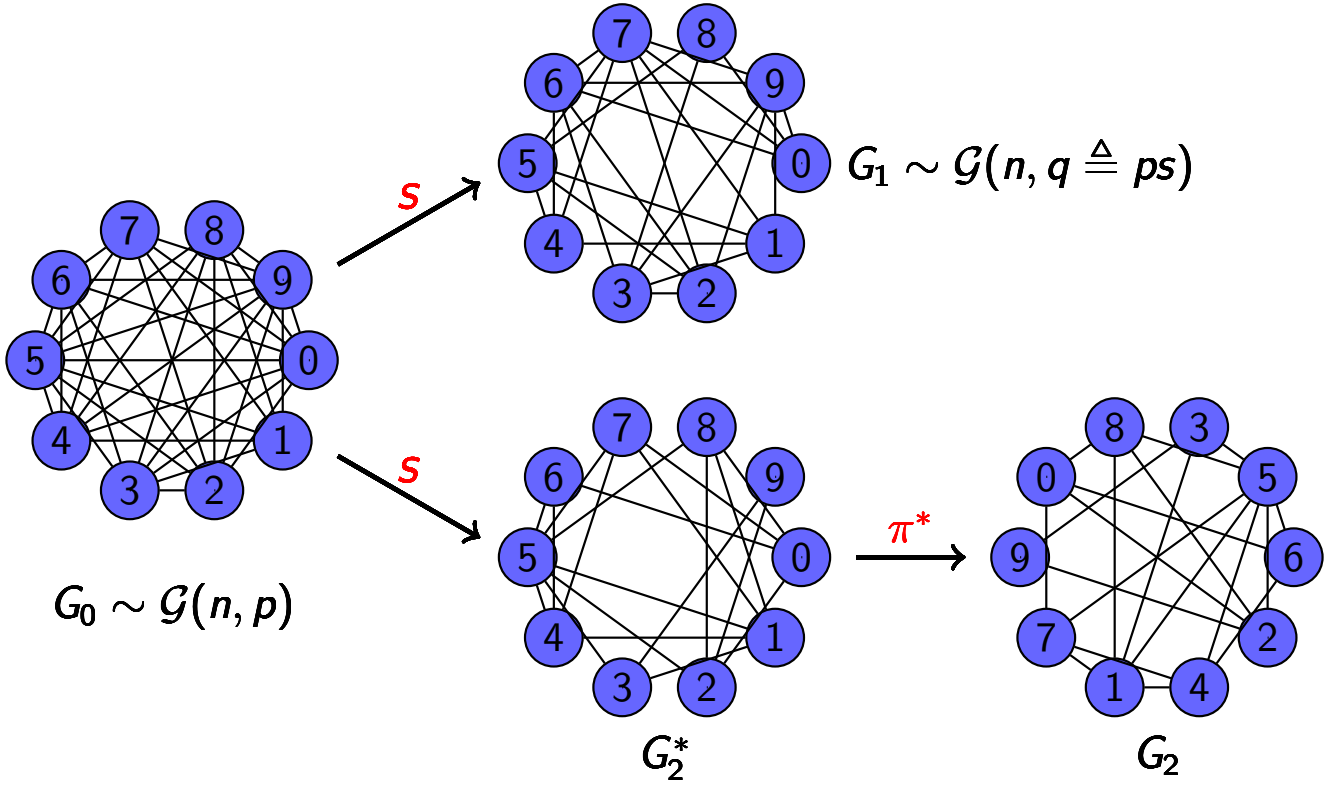

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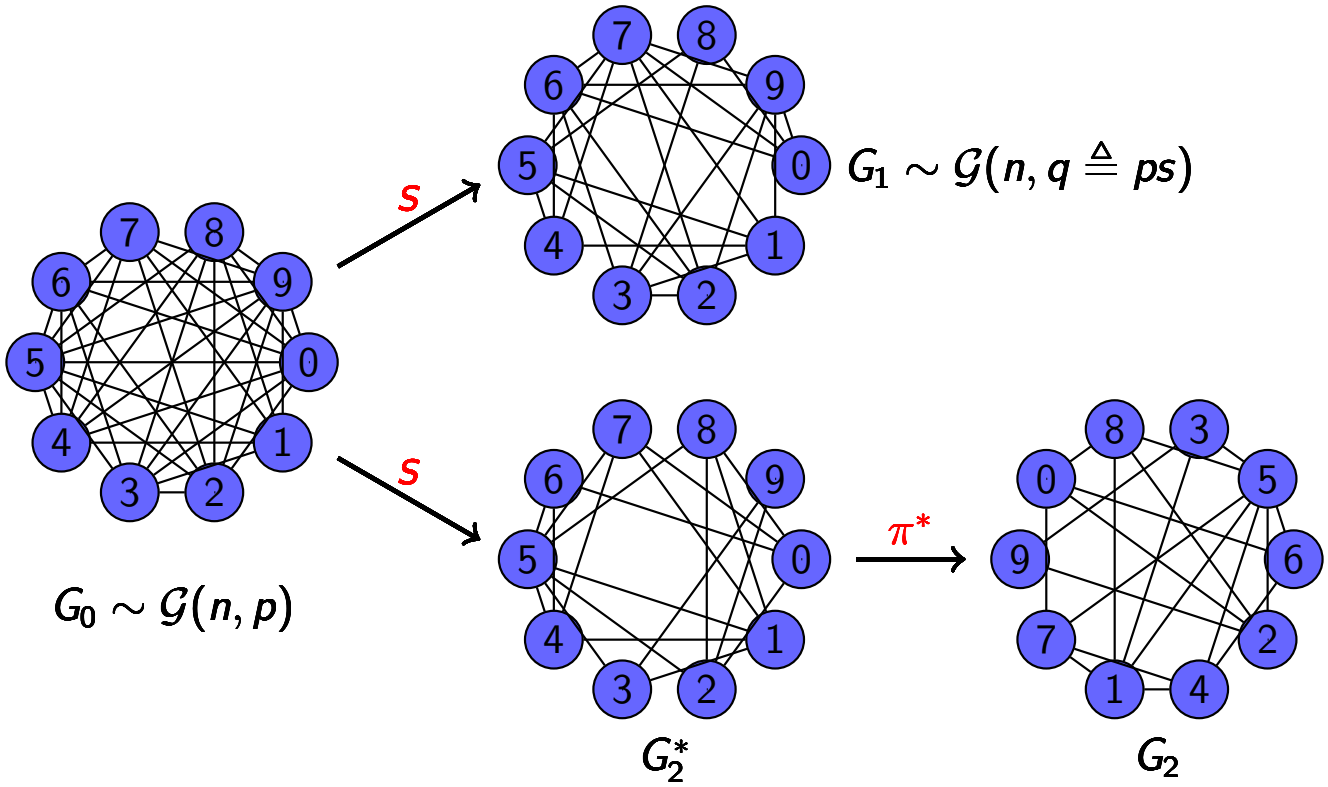

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Disadvantage: almost all realistic networks are not Erdős-Rényi.

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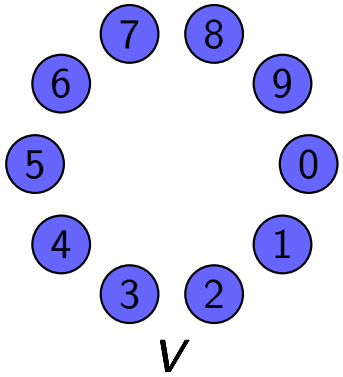

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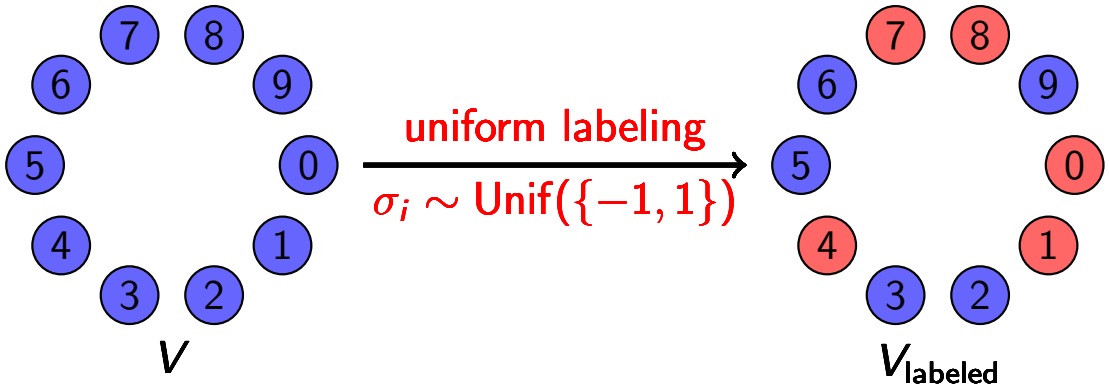

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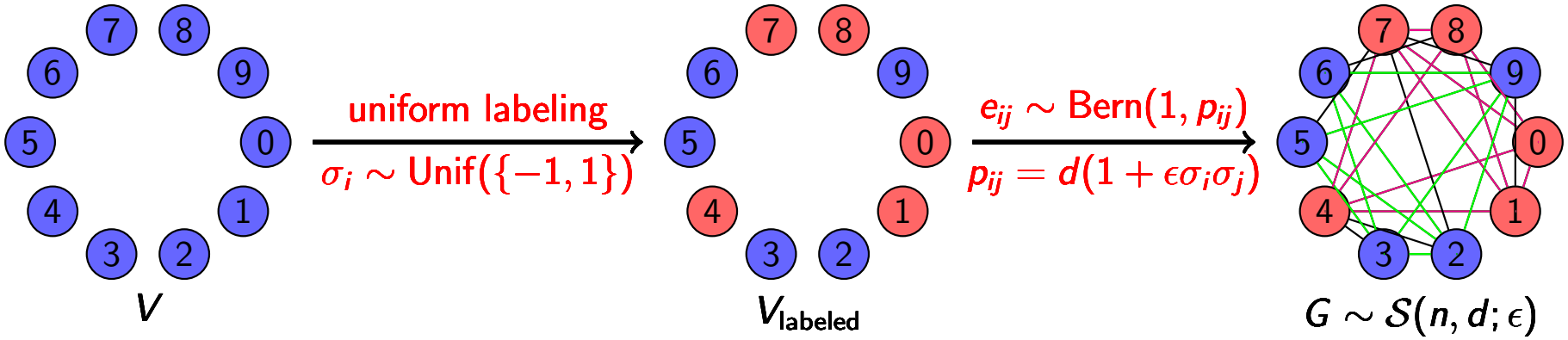

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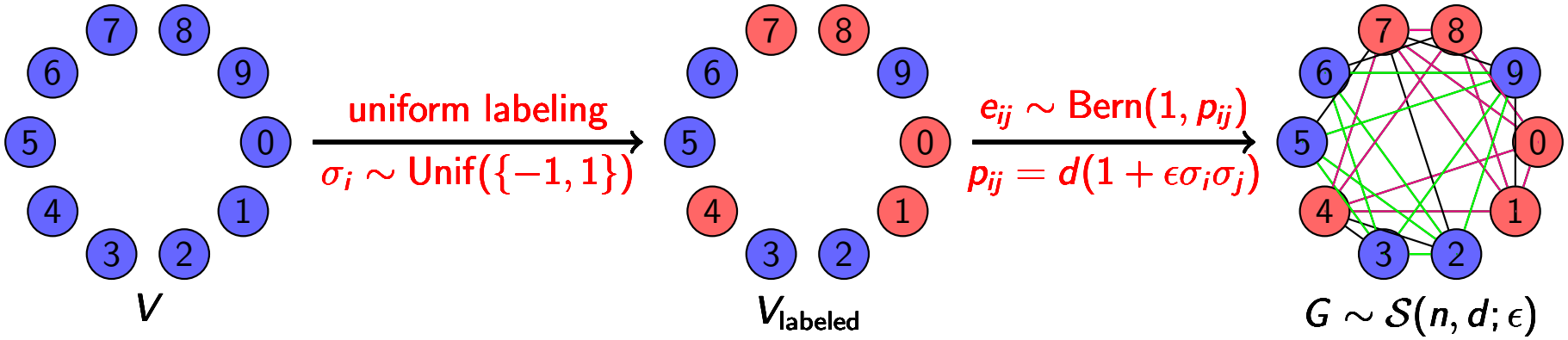

- community structure: an edge joins i and j with probability $p_{ij} = d(1 + \epsilon\sigma_i\sigma_j)$ depending on wether i and j belong to the same community or not.

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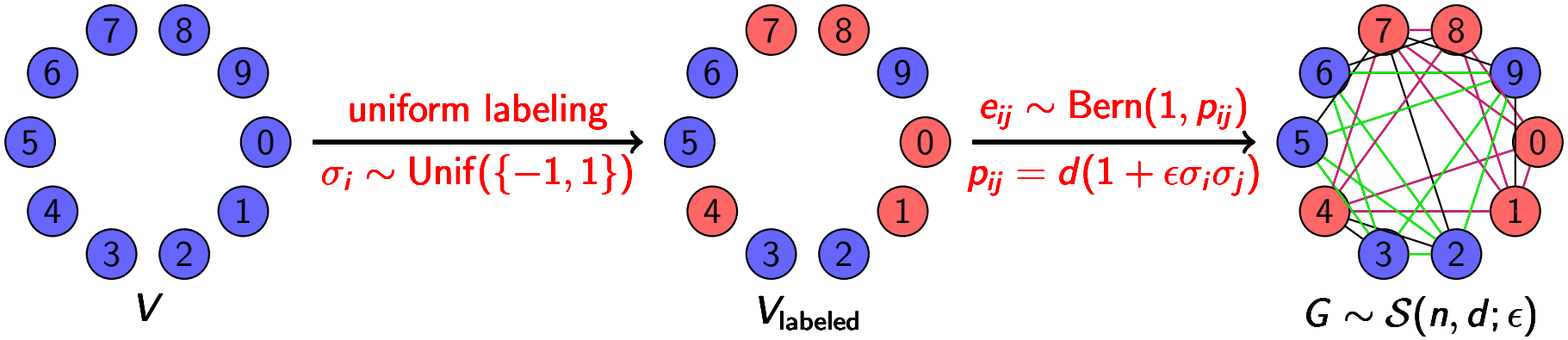

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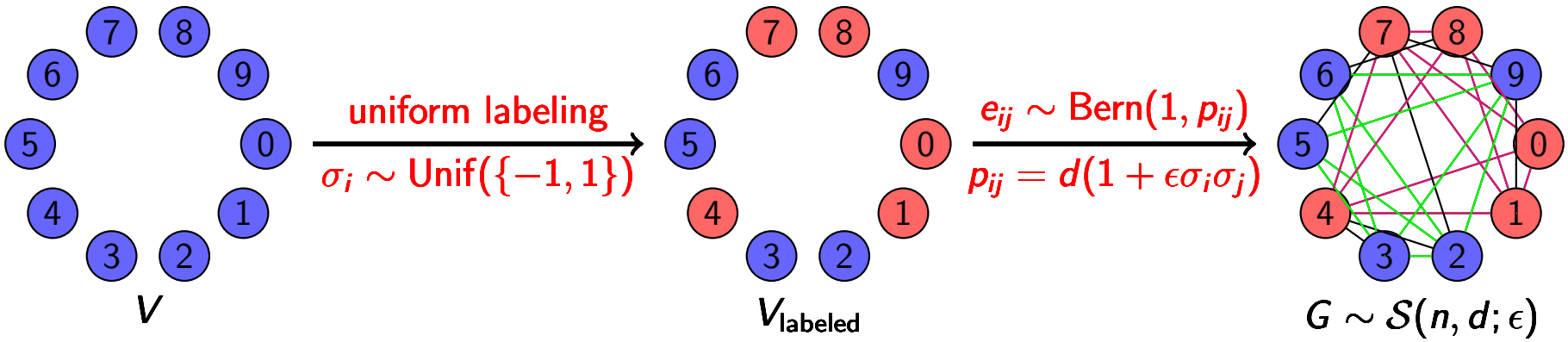

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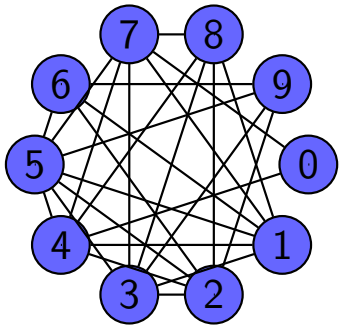
- Generalization to k communities (denoted by $\mathcal{S}(n, d; k, \epsilon)$):
  - $\sigma_i \sim \mathrm{Unif}(\{1, 2, \ldots, k\})$;
  - $p_{ij} = d(1 + \epsilon\omega(\sigma_i, \sigma_j))$ where $\omega(\sigma_i, \sigma_j) = -1 + k \cdot \mathbf{1}_{\{\sigma_i = \sigma_j\}}$.

Correlated stochastic block model



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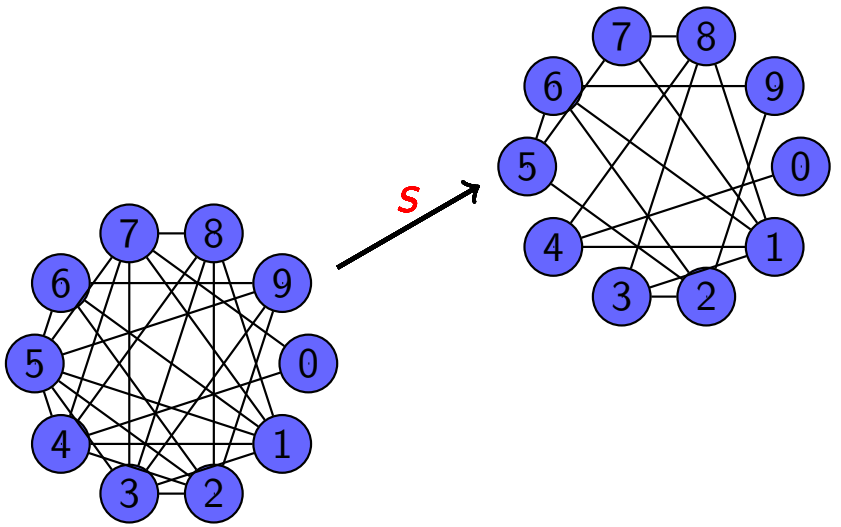

$G_1 \sim \mathcal{S}(n, d' \triangleq ds; k, \epsilon' \triangleq \epsilon s)$

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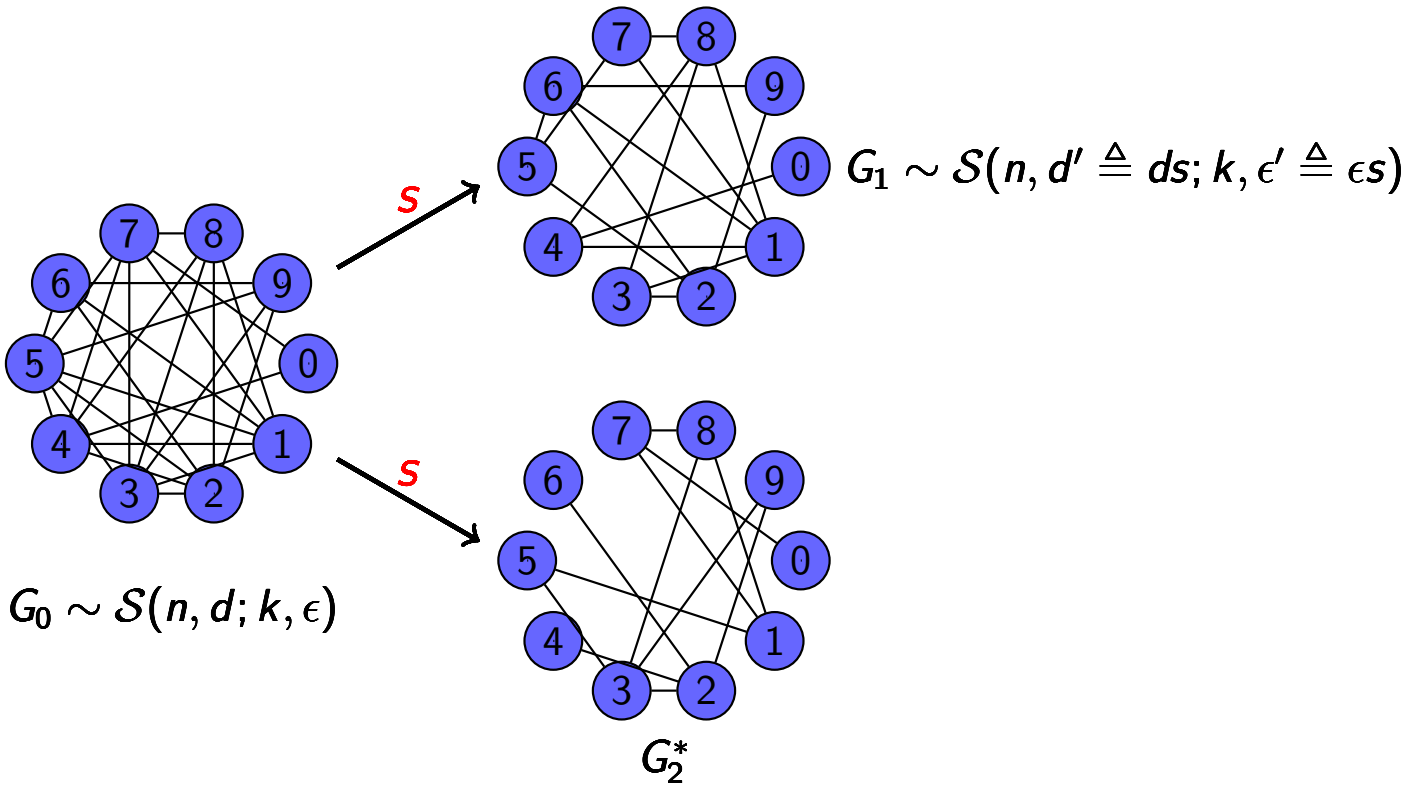

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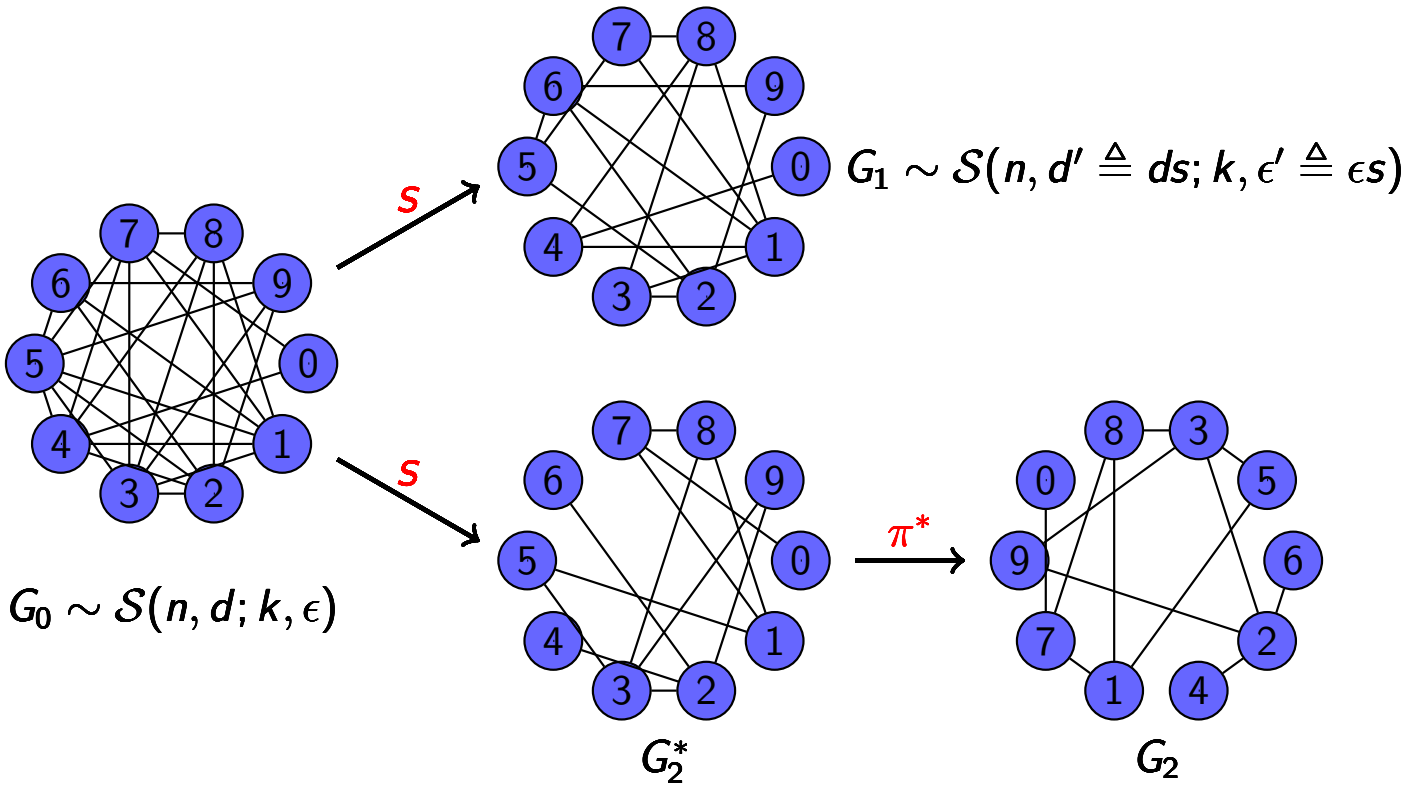

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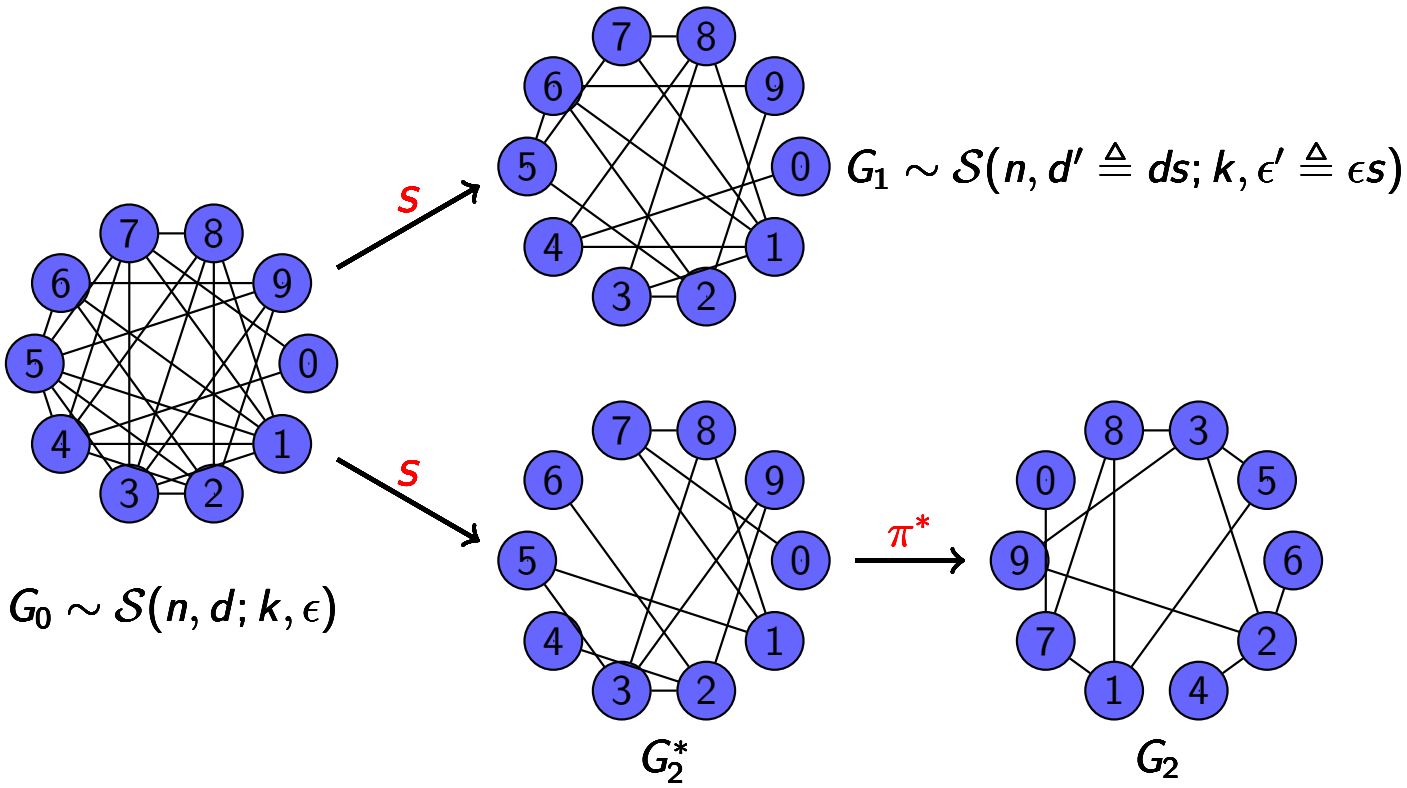

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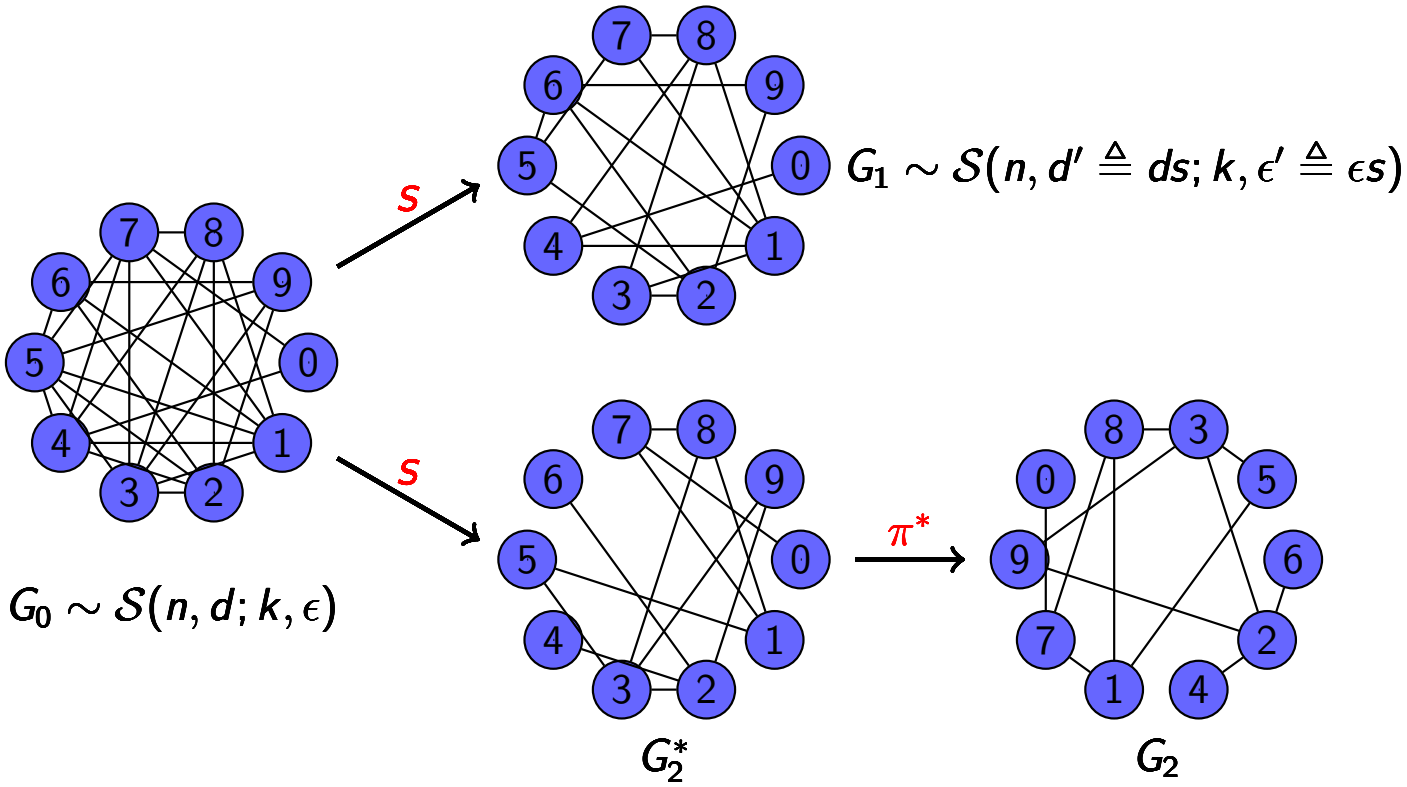

- Generated by independently subsampling $G_0 \sim \mathcal{S}(n, d; k, \epsilon)$ with probability s.
- Denoted by $\mathcal{S}(n, d; k, \epsilon; s)$.

Information threshold: what is the teststone?

Three thresholds: detection, exact recovery, partial recovery.

- Detection: test correlation against independence.
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- D.-Du'23, Du25+: Exact detection and partial recovery threshold in the non-dense regime, via densest subgraph.

Matching algorithms for correlated graphs

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- D.-Li'22+, 23+: poly-time iterative algorithm when correlation is non-vanishing and average degree grows polynomially.

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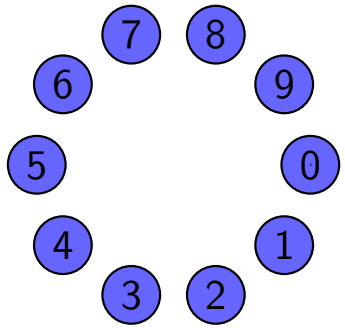
- Application in data privacy: how can we perform a minimal change on the Linkedin and Twitter network, so that it would be computationally hard to recover the matching from the this perturbed observation?
- **Information-computation gap**: a **major** challenge in many random combinatorial optimization and constraint satisfaction problems!

Finding communities in stochastic block model



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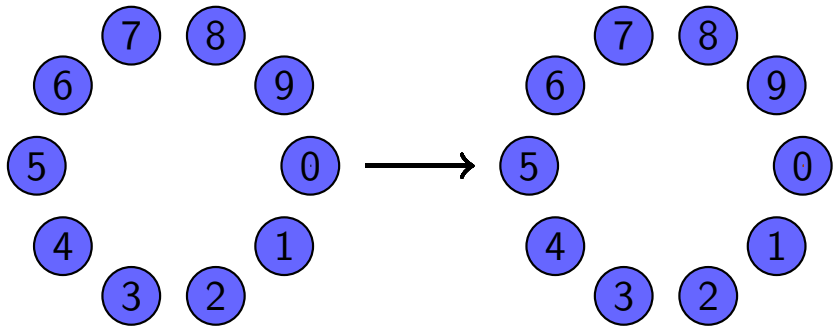

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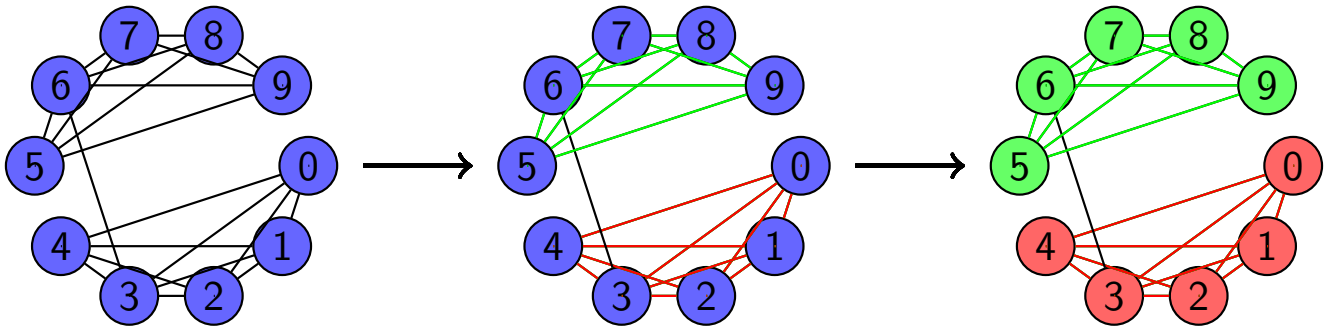

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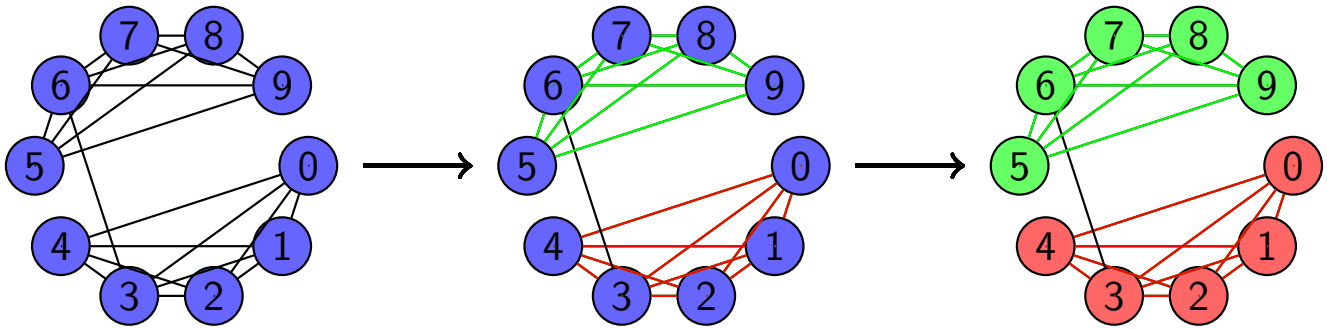

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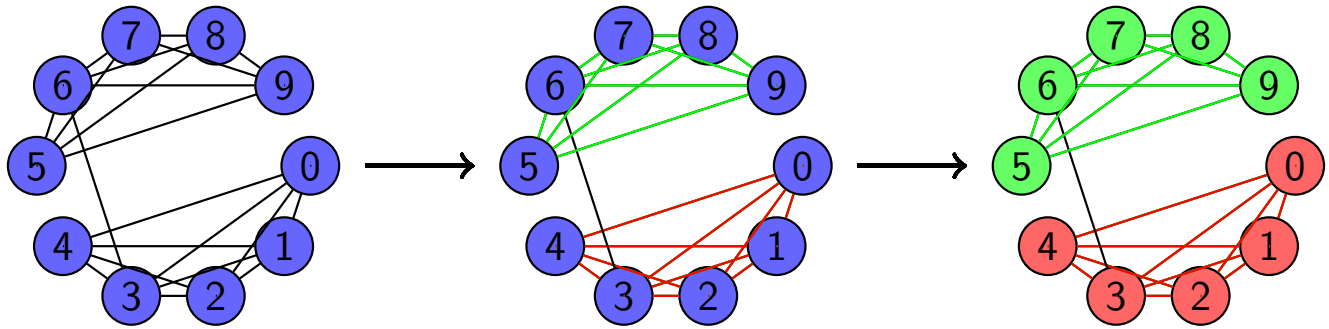

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- Decelle-Krzakala-Moore-Zdeborová'11:
  - Computational transition (under low-degree conjecture) around Kesten-Stigum threshold, i.e., $\lambda\epsilon^2 = 1$ Hopkins-Steurer'16.
  - Presumably information-computation gap if and only if $k \geq 5$ (Mossel-Neeman-Sly'16, Massoulié'16 for $k = 2$ and Mossel-Sly-Sohn'23,24 for general k).

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- Is there any case that we can break the algorithmic barrier in Erdős-Rényi model?

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Using a reduction technique in Li'25+, indicates in the subcritical regime (i.e., when community signal cannot be extracted from a single graph) the detection threshold between correlated/independent block models is still Otter's threshold.

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An ongoing challenge: extending the above result to as long as $\lambda\epsilon^2 > 1$.

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- Bridging what is wanted with what is possible.

Reference: all mentioned works available on arXiv.